Conditions for custodial symmetry in NHDMs

Based on RP, M. Solberg. In preparation

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NHDM Potential

Bilinear form of the NHDM potential

$$V = M_0 K_0 + \Lambda_0 K_0^2 + M_a K_a + L_a K_0 K_a + \Lambda_{ab} K_a K_b\,, \qquad a, b = 1, \ldots, N^2 - 1$$

Under $SU(N)_H$ basis change $\Phi_i \to U_{ij}\Phi_j$

- $M_a \to R(U)_{ab} M_b$
- $L_a \to R(U)_{ab} L_b$
- $\Lambda_{ab} \to R(U)_{ac} R(U)_{bd} \Lambda_{cd}$

with $R(U)$ the adjoint representation

Spectral decomposition of Λ

$$\Lambda = \sum_{a=1}^{N^2-1} \beta_a v_a v_a^T$$

$\to$ **V is determined by**

- $N^2 + 1$ $SU(N)$ invariants
- $N^2 + 1$ adjoint vectors

Identifying symmetries in NHDMs: A challenge

Basis freedom

$$\Phi_i \to U_{ij}\Phi_j \quad \leftrightarrow \quad K_a \to R(U)_{ab}K_b$$

Is there a U such that the symmetry is manifest?

Basis-invariants

- CP: Difficult to find a generating set of CP-odd invariants.
- Applicable to other symmetries $\subset SU(N)_H$

Basis-covariant framework[1]

Basis-invariant properties of basis-covariant objects

Examples:

- Vectors inclusion in subspaces of $\mathbb{R}^{N^2-1}$
- Relative orientation of vectors

$\rightarrow$ **Goal: Relate such properties to the presence of symmetries**

F-product

$\mathbb{R}^{N^2-1}$ is isomorphic to $su(N)$: $v \leftrightarrow V = v_i\lambda_i$ when equipped with so-called F-product

$$F_k^{(u,v)} = f_{ijk}u_iv_j \quad \leftrightarrow \quad [U,V] = 2if_{ijk}u_iv_j\lambda_k$$

$\rightarrow$ We can detect inclusion in a subalgebra

[1] Nishi, Ivanov, Trautner et al. 0605153, 1901.11472, 1903.11110

Custodial symmetry

Manifestly CS NHDM potential

$$\Lambda_C = \begin{pmatrix} C_N & 0 \\ 0 & A_N \end{pmatrix} \qquad \text{where} \quad C_N : k \times k \qquad \left(k \equiv \frac{N(N-1)}{2} \right)$$

with $\begin{pmatrix} C_N & 0 \\ 0 & 0 \end{pmatrix} \equiv \sum_{a=1}^{k} \beta_a t_a t_a^T$ and $\begin{pmatrix} 0 & 0 \\ 0 & A_N \end{pmatrix} t_a = 0$

$$C_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad C_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 & -\alpha & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & -\alpha & 0 & 0 & 0 & 0 \\ \alpha & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Custodial symmetry

Basis-invariant property

Λ has k eigenvectors v_a spanning the subspace $(e_1, ..., e_k) \simeq so(N)$

$\rightarrow$ the set $\{V_a = (v_a)_b \lambda_b\}$

- satisfies $so(N)$ commutation relations
- forms the **defining representation** of $so(N)$

The relevant subspace of $\mathbb{R}^{N^2-1}$ is also a subalgebra!

Can make use of Lie algebra/representation theory to detect custodial symmetry

General necessary and sufficient condition

Sufficient condition

Custodial symmetry $\iff$ Λ has a set of $k = \frac{N(N-1)}{2}$ eigenvectors v_a such that

- Same k eigenvalues as an instance of the block C_N
- Orthogonal to L and M
- Correspond to the **defining representation** of $so(N)$ with the same F-product relations as $\{t_a\}$

Sketch of proof

The representation V_a is equivalent to T_a

$\rightarrow$ there exists $U \in SU(N)$ such that $UV_aU^\dagger = T_a \iff R(U)v_a = t_a$

It follows that $R(U)\Lambda R(U)^T \simeq \Lambda_C$

Can be simplified for N=3 and N=4 Higgs doublets

3HDM[2]

3HDM Custodial block

$$C_3 = 0_{3\times 3}$$

Eigenvectors t_a correspond to the usual $so(3)$ generators **in the defining rep.**

$$2F^{(t_a,t_b)} = \epsilon_{abc} t_c \iff [T_a, T_b] = i\epsilon_{abc} T_c$$

3HDM Custodial symmetry test

Custodial symmetry $\iff$ Λ has 3 orthonormal nullvectors v_a

- $L.v_a = M.v_a = 0$
- $2F^{(v_a,v_b)} = \epsilon_{abc} v_c$

[2] Nishi 2011, 1103.0252

A practical test

Properties

- The numerical prefactor is meaningful (normalized vectors) and excludes the other 3-dimensional reps (e.g. **2+1**)
- $so(3)$ F-products are invariant under rotation of the nullvectors
  $\rightarrow$ if v_a satisfies the condition so does $R_{ab}v_b$

This is a practical test e.g. can be applied numerically

4HDM

Custodial block

$$C_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 & -\alpha & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & -\alpha & 0 & 0 & 0 & 0 \\ \alpha & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvectors

$$t = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Generate the defining rep. of $so(4) \cong so(3) \oplus so(3)$:
$\sqrt{2} F^{(t_a^\pm, t_b^\pm)} = \epsilon_{abc} t_c^\pm$

4HDM

4HDM Custodial symmetry test

Custodial symmetry $\iff$ Λ has 2 sets of 3 orthonormal eigenvectors $v_a^{\pm}$

- eigenvalues $\pm\alpha$
- $L.v_a^{\pm} = M.v_a^{\pm} = 0$
- $\sqrt{2}F^{(v_a^{\pm},v_b^{\pm})} = \epsilon_{abc} v_c^{\pm}$ and $F^{(v_a^{\pm},v_b^{\mp})} = 0$

Properties

- F-product numerical prefactor excludes other 4-dimensional reps. (e.g. **2+2**)
- Based on $so(3)$ F-products $\rightarrow$ invariant under independent rotations of v_a^{+} and v_a^{-}

$N \geq 5$

Difficulties

$$C_N = F(\{\alpha_i\}) \,, \quad i = 1, \ldots, \binom{N}{4}$$

- Eigenvectors t_a depend on α_i for $N \geq 5$
- No eigenvalue pattern for $N \geq 6$

Work in progress

Summary

Take-home messages

- The covariant framework is particularly effective to identify custodial symmetry in NHDM
- In this case eigenvectors span a subalgebra i.e. $so(N)$
- Simple, practical necessary and sufficient condition for $3HDM$ and $4HDM$